Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Further Mathematics
Further Statistics 1
Practice Paper 1

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are **6** questions in this question paper. The total mark for this paper is **75**.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. In a quiz, a team gains 10 points for every question it answers correctly and loses 5 points for every question it does not answer correctly. The probability of answering a question correctly is 0.6 for each question. One round of the quiz consists of 3 questions.

The discrete random variable X represents the total number of points scored in one round. The table shows the incomplete probability distribution of X.

x	30	15	0	-15
$\mathrm{P}(X = x)$	0.216			0.064

(a) Show that the probability of scoring 15 points in a round is 0.432.

(2)

(b) Find the probability of scoring 0 points in a round.

(1)

(c) Find the probability of scoring a total of 30 points in 2 rounds.

(3)

(d) Find $\mathrm{E}(X)$.

(2)

(e) Find $\mathrm{Var}(X)$.

(3)

In a bonus round of 3 questions, a team gains 20 points for every question it answers correctly and loses 5 points for every question it does not answer correctly.

(f) Find the expected number of points scored in the bonus round.

(3)

(Total 14 marks)

[Mark scheme for Question 1](#)

[Examiner comment](#)

2. The cloth produced by a certain manufacturer has defects that occur randomly at a constant rate of λ per square metre. If λ is thought to be greater than 1.5 then action has to be taken.

Using H_0: $\lambda = 1.5$ and H_1: $\lambda > 1.5$ a quality control officer takes a 4 m^2 sample of cloth and rejects H_0 if there are 11 or more defects. If there are 8 or fewer defects she accepts H_0. If there are 9 or 10 defects a second sample of 4 m^2 is taken and H0 is rejected if there are 11 or more defects in this second sample, otherwise it is accepted.

(a) Find the size of this test.

(4)

(b) Find the power of this test when $\lambda = 2$.

(3)

(Total 7 marks)

Mark scheme for Question 2

Examiner comment

3. A company claims that it receives emails at a mean rate of 2 every 5 minutes.

(a) Give two reasons why a Poisson distribution could be a suitable model for the number of emails received.

(2)

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the hypothesis that the mean number of emails received in a 10 minute period is 4. The probability of rejection in each tail should be as close as possible to 0.025.

(2)

(c) Find the actual level of significance of this test.

(2)

To test this claim, the number of emails received in a random 10 minute period was recorded. During this period 8 emails were received.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

During a randomly selected 15 minutes of play in the Wimbledon Men's Tennis Tournament final, 2 emails were received by the company.

(e) Test, at the 10% level of significance, whether or not the mean rate of emails received by the company during the Wimbledon Men's Tennis Tournament final is lower than the mean rate received at other times. State your hypotheses clearly.

(5)

(Total 13 marks)

Mark scheme for Question 3

Examiner comment

4. A number of males and females were asked to rate their happiness under the headings "not happy", "fairly happy" and "very happy".

The results are shown in the table below

		Happiness			Total
		Not happy	Fairly happy	Very happy	
Gender	Female	9	43	34	86
	Male	13	25	16	54
Total		22	68	50	140

Stating your hypotheses, test at the 5% level of significance, whether or not there is evidence of an association between happiness and gender. Show your working clearly.

(Total 10 marks)

Mark scheme for Question 4

Examiner comment

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5. The probability generating function of the random variable X is given by

$$G_X(t) = k(1 + t + 3t^2)^2.$$

(a) Show that $k = \frac{1}{25}$.

(2)

(b) Find $P(X = 2)$.

(2)

(c) Calculate $E(X)$ and $Var(X)$.

(8)

(d) Write down the probability generating function of $2X + 1$.

(2)

(Total 14 marks)

Mark scheme for Question 5

Examiner comment

6. A child is repeatedly twisting a coloured spinner which has a probability 0.4 of landing on red. After each twist the child records whether or not the spinner lands on red.

(a) Show that the probability that the spinner lands on red for the first time occurs on or before the 7th twist is 0.972, to 3 decimal places.
(3)

Find the probability that

(b) exactly three reds occur during the first 7 twists,
(2)

(c) the 3rd red occurs on the 7th twist,
(3)

(d) the 3rd red occurs on or before the 7th twist.
(4)

On another occasion there are 3 children *A*, *B* and *C* playing with the spinner. The children take turns to twist the spinner. Child *A* starts, then *B*, then *C*, then *A* again and so on. The winner is the first child to have the spinner land on red.

(e) Find the probability that *A* wins.
(3)

Given that the first red occurs on or before the 7th twist,

(f) find the probability that *A* wins.
(2)

(Total 17 marks)

Mark scheme for Question 6

Examiner comment

TOTAL FOR PAPER: 75 MARKS

A level Further Mathematics – Further Statistics 1 – Practice Paper 01 – Mark scheme –

Mark scheme for Question 1 (Examiner comment) (Return to Question 1)

Question	Scheme	Marks
1(a)	To score 15 points, 2 correct and 1 not correct	M1
	$[0.6\times0.6\times0.4]+[0.6\times0.4\times0.6]+[0.4\times0.6\times0.6]$ or $3\times(0.6\times0.6\times0.4)$	
	= 0.432 (*)	A1cso
		(2)
(b)	$1-(0.216+0.432+0.064)=$ **<u>0.288</u>** or $3\times0.6\times(0.4)^2$	B1
		(1)
(c)	[(30, 0), (0, 30) or (15, 15)] $0.216\times'0.288'+'0.288'\times0.216+0.432\times0.432$	M1 A1ft
	awrt **<u>0.311</u>**	A1
		(3)
(d)	$\text{E}(X)=[30\times0.216]+[15\times0.432]+[0\times0.288]+[(-15)\times0.064]$	M1
	$\text{E}(X)=12$ **<u>12</u>** (only)	A1
		(2)
(e)	$\text{E}(X^2)=30^2\times0.216+15^2\times0.432+0^2\times0.288+(-15)^2\times0.064\ (=306)$	M1
	$\text{Var}(X)=\text{E}(X^2)-[\text{E}(X)]^2='306'-'12'^2=,$ **<u>162</u>**	M1A1
		(3)
(f)	Let Y = number of points scored in bounce round $[y]$: 60, 35, 10, –15 $[\text{P}(Y=y)]$: 0.216, 0.432, 0.288, 0.064	M1
	$\text{E}(Y)=60\times0.216+35\times0.432+10\times0.288+(-15)\times0.064$	dM1
	$=$ **<u>30</u>**	A1
		(3)
		(14 marks)

Mark scheme for Question 2 (Examiner comment) (Return to Question 2)

Question	Scheme	Marks
2(a)	[X = no. of defects in 4 square metres.] $X \sim \text{Po}(6)$	
	[Size =] $\text{P}(X > 10) + \text{P}(X = 9 \text{ or } 10)\text{P}(X > 10)$	M1
	$= (1 - 0.9574) + (0.9574 - 0.8472)(1 - 0.9574)$	M1A1
	$= 0.04729\ldots$ = awrt **0.0473**	A1
		(4)
(b)	$Y \sim \text{Po}(8)$	B1
	Power $= 1 - (\text{P}(X \leq 8) + [\text{P}(X = 9) + \text{P}(X = 10] \times \text{P}(X \leq 10))$ Or $(1 - \text{P}(X \leq 10)) + [\text{P}(X = 9) + \text{P}(X = 10] \times (1 - \text{P}(X \leq 10))$ $= (1 - 0.8159) + (0.8159 - 0.5925)(1 - 0.8159)$	M1
	$= 0.22522\ldots$ = awrt **0.225**	A1
		(3)
		(7 marks)

Mark scheme for Question 3 (Examiner comment) (Return to Question 3)

Question	Scheme	Marks
3(a)	Any two of	
	• Emails are independent/occur at random	B1
	• Emails occur singly	B1d
	• Emails occur at a constant rate	
		(2)
(b)	$X \sim \text{Po}(4)$	
	$\text{P}(X=0)=0.0183$	
	$\text{P}(X \geq 9)=0.0214$	
	CR $X=0$; $X \geq 9$	B1B1
		(2)
(c)	$0.0183+0.0214=0.0397$ or 3.97%	M1A1
		(2)
(d)	8 is not in the critical region **or** $\text{P}(X \geq 8)=0.0511$	M1
	therefore there is evidence that the company's **claim** is true	A1ft
		(2)
(e)	$\text{H}_0: \lambda=6$ (or $\lambda=2$) $\text{H}_1: \lambda<6$ (or $\lambda=2$) allow λ or μ	B1
	Po(6)	M1
	$\text{P}(X \leq 2)=0.0620$ CR $X \leq 2$	A1
	$0.0620<0.10$	
	Reject H0 or Significant.	M1d
	There is evidence at the 10% level of significance that the mean **rate/number/amount** of **emails** received **is lower/ has decreased/is less.**	A1cso
	Or **fewer emails** are received	
		(5)
		(13 marks)

Mark scheme for Question 4 (Examiner comment) (Return to Question 4)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td rowspan="10">4</td><td>
<table>
<tr><td colspan="2"></td><td colspan="3">Happiness</td></tr>
<tr><td colspan="2"></td><td>Not happy</td><td>Fairly happy</td><td>Very happy</td></tr>
<tr><td rowspan="2">Gender</td><td>Female</td><td>13.51</td><td>41.77</td><td>30.71</td></tr>
<tr><td>Male</td><td>8.49</td><td>26.23</td><td>19.29</td></tr>
</table>
</td><td>M1A1</td></tr>
<tr><td>H_0 : Happiness and gender are independent/ not associated</td><td>B1</td></tr>
<tr><td>H_1 : Happiness and gender are not independent/ associated</td><td>dM1</td></tr>
<tr><td>
<table>
<tr><th>O</th><th>E</th><th>$\frac{(O-E)^2}{E}$</th><th>$\frac{O^2}{E}$</th></tr>
<tr><td>9</td><td>13.51</td><td>1.508</td><td>5.996</td></tr>
<tr><td>43</td><td>41.77</td><td>0.0361</td><td>44.264</td></tr>
<tr><td>34</td><td>30.71</td><td>0.351</td><td>37.637</td></tr>
<tr><td>13</td><td>8.49</td><td>2.402</td><td>19.915</td></tr>
<tr><td>25</td><td>26.23</td><td>0.0575</td><td>23.829</td></tr>
<tr><td>16</td><td>19.29</td><td>0.560</td><td>13.274</td></tr>
</table>
</td><td>A1</td></tr>
<tr><td>$\sum\frac{(O-E)^2}{E} = 4.91$ or $\sum\frac{O^2}{E} - N = 144.91 - 140 = 4.91$</td><td>A1</td></tr>
<tr><td>$\nu = (3-2)(2-1) = 2$</td><td>B1</td></tr>
<tr><td>$\sum\frac{(O-E)^2}{E} < 5.991$</td><td>B1ft</td></tr>
<tr><td>4.91 < 5.991 so ‘insufficient evidence to reject H0 ’ or ‘Accept H0’</td><td>M1</td></tr>
<tr><td>No association between gender and happiness.</td><td>A1</td></tr>
<tr><td></td><td>(10)</td></tr>
<tr><td colspan="3">(10 marks)</td></tr>
</table>

Mark scheme for Question 5 (Examiner comment) (Return to Question 5)

Question	Scheme		Marks
5(a)	$G_x(1) = 1$	Use of $G_x(I) = 1$	M1
	$k = (1 + 1 + 3)^2 = 1$,		
	$k = \frac{1}{25}$ (*)	Fully correct	**A1**
			(2)
(b)	$\frac{1}{25}(1 + t + 3t^2)(1 + t + 3t^2) = \frac{1}{25}(1 + 2t + 7t^2 + \ldots)$		**M1**
	Coefficient of $x^2 = \frac{7}{25}$		**A1**
			(2)
(c)	$G'_X(t) = \frac{2}{25}(1 + 6t)(1 + t + 3t^2)$		**M1A1**
	$G'_X(1) = 2\frac{4}{5}$ $\quad E(X) = 2\frac{4}{5}$	Must say $E(x) = G'_X(1)$	**A1**
	$G''_X(t) = \frac{2}{25}(1 + 6t)(1 + 6t) + \frac{12}{25}(1 + t + 3t^2)$		**M1A1**
	$G''_X(1) = 6\frac{8}{25}$		**A1**
	$\text{Var}(X) = 6\frac{8}{25} + 2\frac{4}{5} - (2\frac{4}{5})^2$		**M1**
	$= 1\frac{7}{25}$		**A1**
			(8)
(d)	$\frac{t}{25}$; $(1 + t^2 + 3t^4)^2$		**B1B1**
			(2)
			(14 marks)

Mark scheme for Question 6 (Examiner comment) (Return to Question 6)

Question	Scheme	Marks
6(a)	X = no. of spin when 1st red occurs $X \sim \text{Geo}(0.4)$	**M1**
	$P(X \leqslant 7) = 1 - P(X > 7) = 1 - (0.6)^7$, $= \mathbf{0.972}$ 3dp *	**M1A1**
		(3)
(b)	Y = no. of reds in 1K 7 spins $Y \sim \text{Bin}(7, 0.4)$ Implied	**M1**
	$P(Y = 3) = \binom{7}{3}(0.4)^3(0.6)^4$ $= \mathbf{0.2903\ or\ 0.290\ or\ 0.29}$	**A1**
		(2)
(c)	R = no. of spin on which 3rd red occurs $R \sim \text{Neg Bin}(0.4, 3)$ Implied	**M1**
	$P(R = 7) = \binom{6}{2}(0.4)^2(0.6)^4 \times 0.4$, $= 0.1244$ or 0.124	**M1A1**
		(3)
(d)	$P(R \leqslant 7) = (c) + P(R = 6) + P(R = 5) + P(R = 4) + P(R = 3)$	**M1**
	$= (0.4)^3[15 \times (0.6)^4 + 10 \times (0.6)^3 + 6 \times (0.6)^2 + 3 \times 0.6 + 1]$	**M1A1**
	$= 0.580096..$ $= \mathbf{0.58, 0.580, 0.5801}$	**A1**
		(4)
(e)	$P(A \text{ wins}) = 0.4 + (0.6)^3 \times 0.4 + (0.6)^6 \times 0.4 + \ldots$	**M1**
	$= \dfrac{0.4}{1-(0.6)^3}$, $= \mathbf{0.510\ or\ 0.51\ or\ 0.5102}$	**M1A1**
		(3)
(f)	$P(A \text{ wins} \mid X \leqslant 7) = \dfrac{(0.4 + (0.6)^3 \times 0.4 + (0.6)^6 \times 0.4)}{(a)}$, $= \mathbf{0.5196\ or\ 520}$	**M1A1**
		(2)
		(17 marks)

A level Further Mathematics – Further Statistics 1 – Practice Paper 01 – Examiner report –

Examiner comment for Question 1 **(Mark scheme)** **(Return to Question 1)**

1. In part (a) many realised that a score of 15 came from getting 2 correct answers and one incorrect answer but they just gave the probability as $0.6 \times 0.6 \times 0.4$ omitting the multiplication by 3. Others thought that the probability of 0.432 came from 2×0.216 and scored zero. Most obtained 0.288 in part (b) though usually, though not exclusively, by using the fact that the sum of the probabilities equalled 1. Some candidates argued in circles, using the given value of 0.432 to find 0.288 and then using their derived value of 0.288 to "find" 0.432, they of course, scored no marks for part (a). Part (c) proved to be quite discriminating. Many could identify some of the required cases and 0.216×0.288 was often seen or sometimes 0.432^2 but it was less common to see all 3 cases included. Some attempted to consider the situation as a set of 6 questions but they rarely considered all ${}^6C_4 = 15$ arrangements. There was some evidence that candidates were mis-interpreting "a total of 30 points in 2 rounds" to mean "30 points in each of 2 rounds" and giving their answer as simply 0.216^2 : they should be encouraged to read and interpret the questions carefully. The methods for part (d) and (e) were well known and many scored well here. A number failed to use brackets carefully and found $-15^2 \times 0.064$ rather than $(-15)^2 \times 0.064$ but there were fewer cases than sometimes of candidates forgetting to square the mean before subtracting or thinking that $\text{Var}(X) = \text{E}(X^2)$ in part (e). In the final part most chose to form the distribution of Y = the number of points in a bonus round and hence find E(Y) using the given formula. A common mistake was to have 0 instead of 10 and a few candidates gave an answer of 35 since this value had the highest probability. A handful of candidates spotted that $Y = \frac{5}{3}X + 10$ and were able to write down the answer quite simply but this was very rare.

Examiner comment for Question 2 **(Mark scheme)** **(Return to Question 2)**

2. Part (a) was generally well answered but part (b) proved to be quite challenging for candidates. The main errors were to work out the probability of a type II error rather than work out the power and not realising the probability of accepting a sample changes for the second sample.

Examiner comment for Question 3 **(Mark scheme)** **(Return to Question 3)**

3. In part (a) many students had learnt when a Poisson distribution is suitable but too many did not use the context given in the question. It was pleasing to see that few students gave independent and random as separate reasons although some students lost the mark for "constant rate" by talking vaguely about rate or giving the conditions needed when approximating to a normal distribution.

   Most students knew how to set about finding a critical region in part (b) and located the correct probabilities but gave the incorrect critical regions with $X \geq 8$ being the most common error although $X \geq 10$ and$X \geq 7$ were occasionally seen. Fewer students, than in previous years, gave the critical region using probability statements although a small minority gave critical values rather critical regions.

   Part (c) was usually correct even if their answers to part (b) had failed to gain full credit.

   In part (d) the majority of students stated whether 8 was or was not in their critical region and drew the correct contextual conclusion and managed to associate the "claim" with H_0 rather than H_1. A few had not appreciated the connection with part (b) and redid the entire calculation. A minority of students did not appreciate that a statistical justification was required and said that 8 was double the expected number of emails and so was a significant result.

Part (e) was well done, even by students who had gained few marks up to this point. The first 3 marks were usually scored, although calculating $\mathrm{P}(X = 2)$ or $\mathrm{P}(X \leq 1)$ were sometimes seen. A minority wrongly accepted H_0 and did not include enough context their conclusion. A mistake sometimes seen was to compare the probability with 0.05 and incorrectly accept the null hypothesis.

Examiner comment for Question 4 (Mark scheme) (Return to Question 4)

4. This was a routine question for the majority of students. Both the hypotheses and the result of the test were usually given in context and many fully correct solutions were seen here. Premature rounding proved costly in some solutions offered and typically these students did not show all their calculations

Examiner comment for Question 5 (Mark scheme) (Return to Question 5)

5. This question was very well done with the majority of candidates gaining full marks.

Examiner comment for Question 6 (Mark scheme) (Return to Question 6)

6. Candidates often confused the distributions in this question, with the answer to part (c) often offered in part (b). The more complex probabilities later on in the question defeated many; although even some weak candidates were able identify the geometric progression in part (e) successfully.